

The average velocity of self-propelled particles in a two-dimensional potential with colored noise

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Abstract

The average velocity of self-propelled particles in a two-dimensional potential with colored noise is investigated. The current reversal phenomenon appear with changing x direction colored noise intensity. There exist optimal values of the parameters at which the average velocity takes its maximal value. The y direction noise and the self-propelled angle noise have great effects on the x direction average velocity, but they can not induce x direction particles transport phenomenon themselves.

Keywords: Self-propelled Particles, Average Velocity, Current Reversal

1. Introduction

The transport of particles induced by zero average fluctuations and random perturbations plays a crucial role in many physical and biological systems, which has many theoretical and practical implications[1, 2, 3, 4]. Some Brownian ratchets have been proposed to model the unidirectional motion driven by nonequilibrium fluctuations and random perturbations, such as diffusion ratchets, rocking ratchets, flashing ratchets, and so on. These previous ratchet models mainly focus on passive Brownian particles. Recently, works of active Brownian particles have received some attention[5, 6]. Self-propelled particles have been investigated as the simplest model for "active materials", and the kinetic of self-propelled particles moving in potential could exhibit peculiar behavior[7, 8, 9, 10, 11, 12].

There are numerous realizations of self-propelled particles in nature ranging from bacteria and spermatozoa to artificial colloidal microswimmers. D'Orsogna *et al.* modeled a self-propelling biological or artificial individuals

interacting through pairwise attractive and repulsive forces, and predicted stability and morphology of organization starting from the shape of the two-body interaction[7]. H. Chaté *et al.* investigated Vicsek-style self-propelled particle models and observed a statistically homogeneous ordered phase[9]. Angelani *et al.* studied the run and tumble particles in periodic potential and found the asymmetric potential produces a net drift speed[13, 14]. Wan *et al.* showed a rectification phenomenon which is similar to experimental results achieved in a simple model for over damped swimming bacteria[15]. Ghosh *et al.* studied the transport of Janus particles in periodically compartmentalized channel and found that the rectification can be orders of magnitude stronger than that for ordinary thermal potential ratchets[16]. Potosky *et al.* found that even in a symmetric potential a spatially modulated self-propelled velocity can induce the directed transport[17]. Hagen *et al.* studied the over damped Brownian motion of a self-propelled particle by solving the Langevin equation analytically and calculated the first four moments of the probability distribution function[18]. In all these studies, the potential is quasi-one-dimensional and independent in each dimension.

In 2014, Ai *et al.* investigated the rectification and diffusion of self-propelled particles in a two-dimensional corrugated channel with white noise, and found the self-propelled velocity can strongly increase the effective diffusion, while the large rotational diffusion rate can strongly suppress the effective diffusion[22]. In this paper, we extended these studies to the case of a two-dimensional potential with colored noise. The paper is organized as follows: In Section 2, the basic model of self-propelled ratchets with a two-dimensional potential and colored noise is provided. In Section 3, the effects of parameters is investigated by means of simulations. In Section 4, we get the conclusions.

2. Basic model and methods

In the present work, we attempt to consider self-propelled particles that move in a two dimensional potential. The dynamics of Brownian particle is governed by the following Langevin equations[19, 20].

$$\frac{dx}{dt} = v_0 \cos \theta + \mu F_x + \xi_1(t), \quad (1)$$

$$\frac{dy}{dt} = v_0 \sin \theta + \mu F_y + \xi_2(t), \quad (2)$$

$$\frac{d\theta}{dt} = \xi_3(t), \quad (3)$$

v_0 is the self-propelled velocity and μ is the mobility. θ is the self-propelled angle. $F_x = -\frac{\partial U(x,y)}{\partial x}$ and $F_y = -\frac{\partial U(x,y)}{\partial y}$.

$$U(x, y) = -U_0 \sin\left(\frac{2\pi x}{L}\right) + \frac{1}{2}C_0[1 - \lambda \sin\left(\frac{2\pi x}{L} + \phi\right)]y^2, \quad (4)$$

is the potential, which periodic in x direction and parabolic in y direction[21]. U_0 is the height of the x direction potential. C_0 is the intensity of the y direction potential. ϕ is the phase shift between the x direction potential and the modulation function. λ is the modulation constant with $0 < \lambda < 1$. ξ_1 is the x direction Gaussian colored noise. ξ_2 is the y direction Gaussian colored noise. ξ_3 is the self-propelled angle colored noise, and describes the nonequilibrium angular fluctuation. $\xi_{1,2,3}$ satisfies the following relations:

$$\langle \xi_i(t) \rangle = 0, \quad (5)$$

$$\langle \xi_i(t) \xi_j(s) \rangle = \delta_{ij} \frac{Q_i}{\tau_i} \exp\left[-\frac{|t-s|}{\tau_i}\right], i, j = 1, 2, 3, \quad (6)$$

$\langle \dots \rangle$ denotes an ensemble average over the distribution of the random forces. $Q_1(Q_2)$ is the $x(y)$ direction colored noise intensity. Q_3 is the colored noise intensity of self-propelled angle. τ_1 , τ_2 and τ_3 are the self-correlation time of the noises, respectively. Upon introducing characteristic length scale L , time scale $\tau_0 = \frac{L^2}{\mu U_0}$, and energy U_0 , Eqs. (1, 2, 3) can be rewritten in dimensionless form:

$$\frac{d\hat{x}}{d\hat{t}} = \hat{v}_0 \cos \theta + \hat{F}_{\hat{x}} + \hat{\xi}_1(\hat{t}), \quad (7)$$

$$\frac{d\hat{y}}{d\hat{t}} = \hat{v}_0 \sin \theta + \hat{F}_{\hat{y}} + \hat{\xi}_2(\hat{t}), \quad (8)$$

$$\frac{d\theta}{d\hat{t}} = \hat{\xi}_3(\hat{t}), \quad (9)$$

with $\hat{x} = \frac{x}{L}$, $\hat{y} = \frac{y}{L}$, $\hat{t} = \frac{t}{\tau_0}$, $\hat{U} = \frac{U}{U_0}$, $\hat{v}_0 = \frac{v_0 L}{\mu U_0}$, $\hat{Q}_1 = \frac{Q_1}{\mu U_0}$, $\hat{Q}_2 = \frac{Q_2}{\mu U_0}$, $\hat{Q}_3 = \frac{Q_3 L^2}{\mu U_0}$, $\hat{\tau}_1 = \frac{\tau_1}{\tau_0}$, $\hat{\tau}_2 = \frac{\tau_2}{\tau_0}$, $\hat{\tau}_3 = \frac{\tau_3}{\tau_0}$. The dimensionless form of potential(show in Fig.(1)) is rewritten as

$$\hat{U}(\hat{x}, \hat{y}) = -\sin(2\pi\hat{x}) + \frac{1}{2}\hat{C}_0[1 - \lambda \sin(2\pi\hat{x} + \phi)]\hat{y}^2 \quad (10)$$

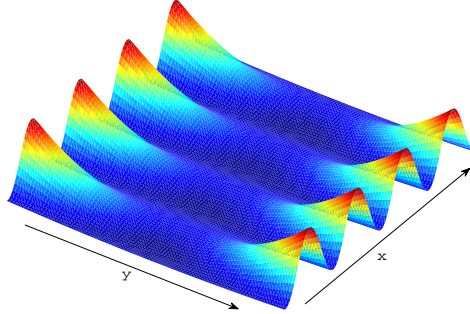


Figure 1: The two dimensional periodic potential $U(x,y)$, which is periodic in x direction and parabolic in y direction.

with $\hat{C}_0 = \frac{C_0 L^2}{U_0}$. In this paper, we will use only the dimensionless Eqs. (7,8,9) and shall omit the hat(\wedge) for all quantities[22].

A central practical question in the theory of Brownian motors is the over all long-time behavior of the particle, and the key quantities of particle transport through periodic potential is the particle velocity V . V can be corroborated by Brownian dynamic simulations performed by integration of the Langevin equations using the stochastic Euler algorithm. We only calculate the x direction average velocity as the potential along y direction is parabolic. The x direction average velocity can be obtained from the following equation:

$$V = \lim_{t \rightarrow \infty} \frac{\langle x(t) - x_0 \rangle}{t - t_0} \quad (11)$$

$x(t)$ is the position of particles at time t , and $x(t_0) = x_0$.

3. Results and discussion

In order to give a simple and clear analysis of the system, Eqs.(7), (8) and (9) are integrated using the Euler algorithm with $C_0 = 5.0$, $\phi = \frac{\pi}{2}$ and time step $\Delta t = 10^{-4}$. The total integration time was more than 1.5×10^6 . The stochastic averages reported above were obtained as ensemble averages over 3×10^4 trajectories with random initial conditions.

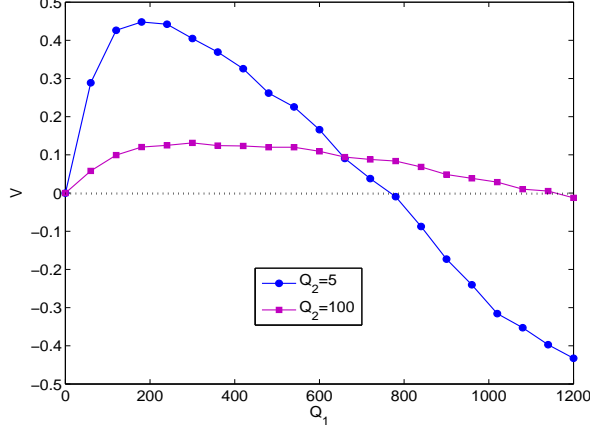


Figure 2: Average velocity V as a function of the x direction noise intensity Q_1 with different Q_2 . The other parameters are $Q_3 = 0.01$, $\lambda = 0.9$, $v_0 = 2$ and $\tau_1 = \tau_2 = \tau_3 = 1$.

The average velocity V as a function of the x direction noise intensity Q_1 is reported in Fig.2. We find that there exist a maximum with increasing Q_1 , which shows the feature of resonance. The smaller y direction noise intensity Q_2 , the resonance phenomenon is more obvious. As Q_1 is small, $V \rightarrow 0$, the particle stays at the bottom of the potential and can not pass the barrier. We also find, for $Q_2 = 5$, $V < 0$ as $Q_1 > 800$, so the current reversal phenomenon appear with increasing Q_1 .

Figure 3 shows the dependence of V on Q_1 with different self-propelled angle noise intensity Q_3 . Just like Fig.(2), V has a maximum with increasing Q_1 . We find the peak of the curve moves to left with decreasing Q_3 , and the smaller Q_3 , the higher of the peak is.

Average velocity V as a function of y direction noise intensity Q_2 with different Q_1 is shown in Fig.(4). V has a marked peak with increasing Q_2 as Q_1 is large($Q_1 = 100$), and V has always been zero with increasing Q_2 as Q_1 small($Q_1 = 5$). So x direction noise Q_1 can induce x direction particles transport, and an optimal value of y direction noise Q_2 appear at which V takes its maximal value. As Q_1 is small, the particle stays at the bottom of the potential and can not transport the potential regardless of Q_2 large or small. x direction noise can induce x direction particles transport, and y direction noise have some effects on the particles transport phenomenon, but y direction noise can not induce x direction particles transport as x direction noise is small.

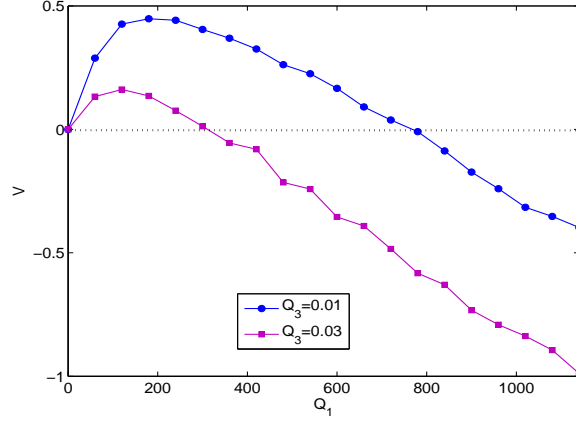


Figure 3: Average velocity V as a function of Q_1 with different Q_3 . The other parameters are $Q_2 = 5$, $\lambda = 0.9$, $v_0 = 2$ and $\tau_1 = \tau_2 = \tau_3 = 1$.

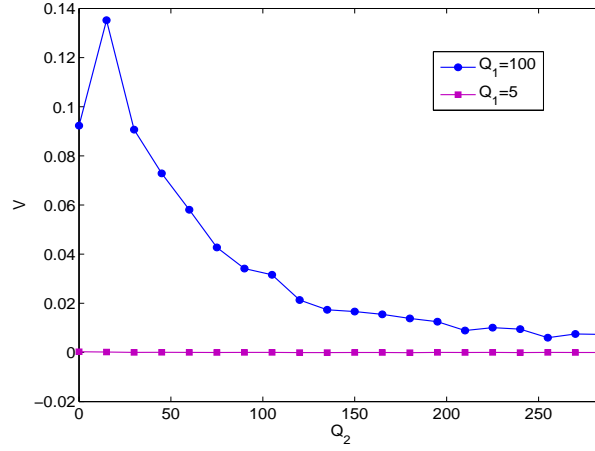


Figure 4: Average velocity V as a function of the y direction noise intensity Q_2 with different Q_1 . The other parameters are $Q_3 = 0.03$, $\lambda = 0.9$, $v_0 = 2$ and $\tau_1 = \tau_2 = \tau_3 = 1$.

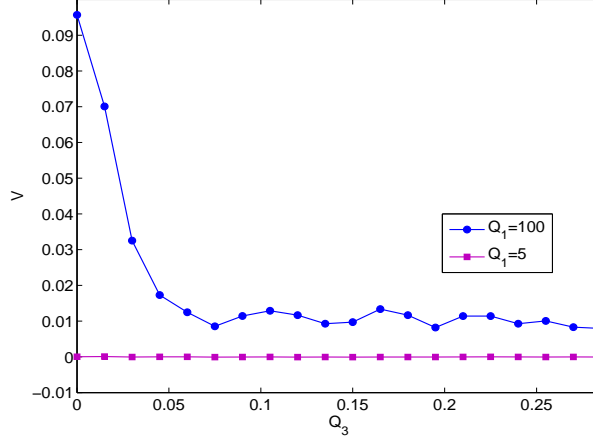


Figure 5: Average velocity V as a function of the noise intensity Q_3 with different Q_1 . The other parameters are $Q_2 = 100$, $\lambda = 0.9$, $v_0 = 2$ and $\tau_1 = \tau_2 = \tau_3 = 1$.

Figure 5 shows the dependence of V on the self-propelled angle noise intensity Q_3 with different Q_1 . As $Q_1 = 100$, we find V appears periodic fluctuations and the amplitude becomes smaller and smaller with increasing Q_3 . V has always been zero with increasing Q_3 as Q_1 is small ($Q_1 = 5$). So as Q_1 is large, small Q_3 should maximize the x direction velocity V , and self-propelled angle noise can not induce x direction particles transport itself as Q_1 is small. From Fig.(4) and Fig.(5), we find Q_2 and Q_3 can not induce particles transport themselves, but they have great effects on Q_1 induced x direction particles transport.

Figure 6 shows the dependence of V on the the self-correlation time τ_1 . It is found V decreases monotonically with increasing τ_1 . So large τ_1 cause the inhibition of the particles transport.

Average velocity V as a function of self-correlation time τ_2 is shown in Fig.(7). Unlike Fig.(5), we find V increase with increasing τ_2 , so large τ_2 is better for particles transport.

Figure 8 depicts average velocity V as a function of the self-correlation time τ_3 . We can find V appear periodic fluctuations with increasing τ_3 , and the amplitude becomes smaller and smaller with increasing τ_3 .

Average velocity V as a function of the modulation constant λ is shown in Fig.9. As the modulation constant λ increases, the average velocity first increases, and then decreases near $\lambda = 0.5$. There exists an optimal value of λ at which the average velocity V takes its maximal value. When there is no

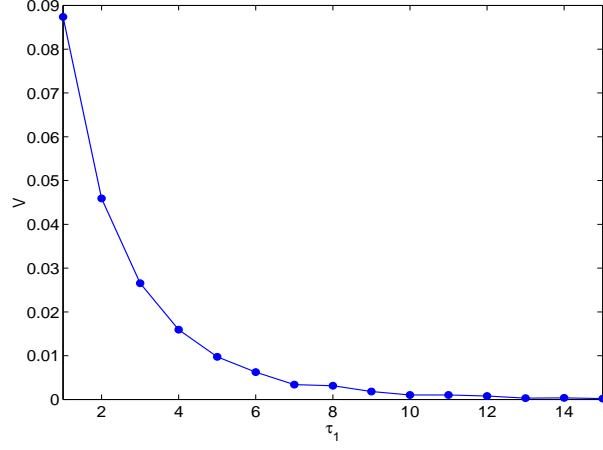


Figure 6: Average velocity V as a function of the self-correlation time τ_1 . The other parameters are $Q_1 = Q_2 = 100$, $Q_3 = 0.03$, $\lambda = 0.9$, $v_0 = 2$ and $\tau_2 = \tau_3 = 1$.

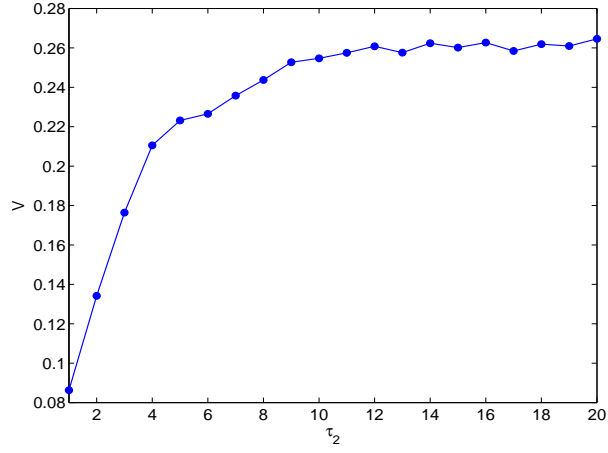


Figure 7: Average velocity V as a function of the self-correlation time τ_2 . The other parameters are $Q_1 = Q_2 = 100$, $Q_3 = 0.03$, $\lambda = 0.9$, $v_0 = 2$ and $\tau_1 = \tau_3 = 1$.

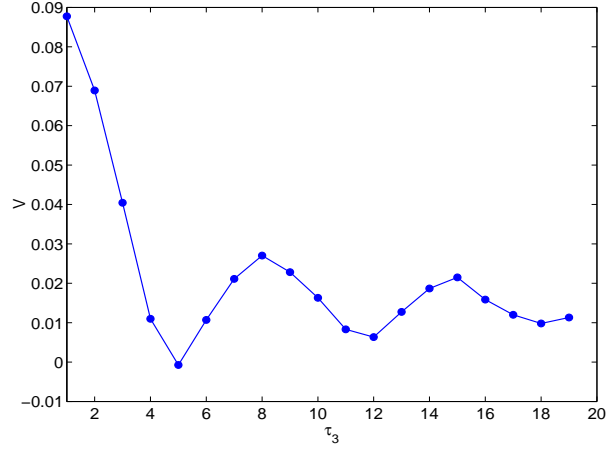


Figure 8: Average velocity V as a function of the self-correlation time τ_3 . The other parameters are $Q_1 = Q_2 = 100$, $Q_3 = 0.03$, and $\tau_1 = \tau_2 = 1$.

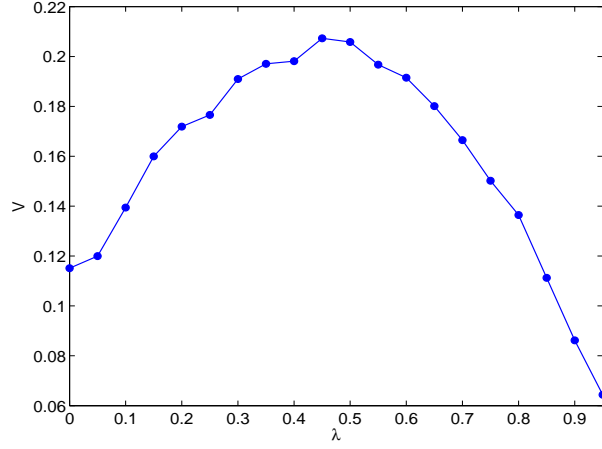


Figure 9: Average velocity V as a function of the modulation constant λ . The other parameters are $Q_1 = Q_2 = 100$, $Q_3 = 0.03$, $v_0 = 2$ and $\tau_1 = \tau_2 = 1$.

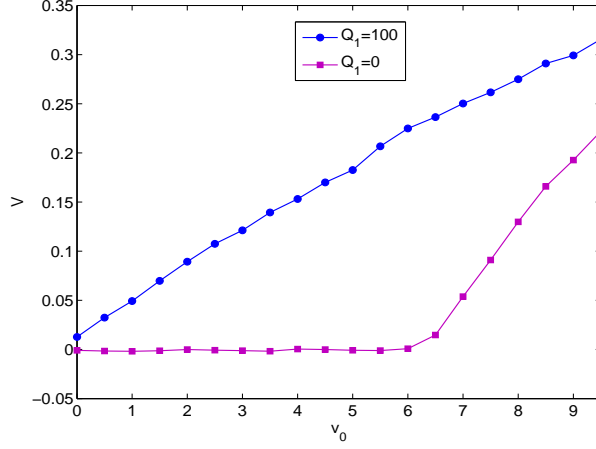


Figure 10: Average velocity V as a function of the self-propelled velocity v_0 with different Q_1 . The other parameters are $Q_2 = 100$, $Q_3 = 0.03$, and $\tau_1 = \tau_2 = \tau_3 = 1$.

coupling between the x direction potential and the y direction potential ($\lambda = 0$), the average velocity $V \neq 0$. The results differ from the results of Ref.[22], the major reason is the independent of noise ξ_1 and ξ_2 in our study, and ξ_1 and ξ_2 have different effects on the system.

Figure 10 depicts V as a function of the self-propelled velocity v_0 with different Q_1 . As $Q_1 = 100$, we find $V \neq 0$ as $v_0 = 0$, so proper ξ_1 can induce particles transport itself whether the self-propelled velocity exists or not. As $Q_1 = 0$, it is found that the average velocity is zero until $v_0 > 6$, the reason is the value of the x direction force for the potential is $|\hat{F}_x| = |-\frac{\partial \hat{U}(x,y)}{\partial \hat{x}}| = |2\pi \cos(2\pi \hat{x})| \leq 2\pi$. Therefore, in order to pass through the channel, the particle needs a nonzero v_0 to move against the max force $F_{x(max)} = 2\pi$. This result is the same as the result of Ref.[22].

4. Conclusions

In this paper, we numerically studied the transport of self-propelled particles in a two-dimensional potential with colored noise. We find that the average velocity V has a maximum with increasing x direction diffusion constant Q_1 . Large Q_1 can induce current reversal phenomenon. y direction diffusion constant and the rotational diffusion rate Q_3 can not induce x direction particles transports themselves, but they have great effects on x direction average velocity V . V has a maximum with increasing Q_2 and ap-

pear periodic fluctuations with increasing Q_3 . V decreases with increasing τ_1 (self-correlation time of x direction noise) and increases with increasing τ_2 (self-correlation time of y direction noise). V appear periodic fluctuations with increasing self-correlation time τ_3 .

5. Acknowledgments

Project supported by Natural Science Foundation of Anhui Province(Grant No:1408085QA11) and the National Natural Science Foundation of China (No:11404005).

References

- [1] P.Hänggi and F. Marchesoni, Rev. Mod. Phys. 81 (2009) 387.
- [2] J. Spiechowicz, P. Hänggi, and J. Luczka1, Phys. Rev. E 90 (2014) 032104.
- [3] X. Zhang and B. Ai,J. Phys. A: Math. Theor. 43 (2010) 495004.
- [4] L. Du and D. Mei, Phys. Rev. E 90 (2012) 032104.
- [5] J. Toner, Y. Tu, and S. Ramaswamy, Annals of Physics 318 (2005) 170.
- [6] M. E. Cates, J. Tailleur, EPL 101 (2013) 20010.
- [7] M. R. D’Orsogna, Y. L. Chuang, A. L. Bertozzi and L. S. Chayes, Phys. Rev. Lett.96 (2006) 104302.
- [8] A. Czirók and T. Vicsek, Physica A 281 (2000) 17.
- [9] H. Chaté, F. Ginelli, G. Grégoire, and F. Raynaud, Phys. Rev. E 77 (2008) 046113.
- [10] J. D. Bao, Phys. Rev. E 63 (2001) 061112.
- [11] J. D. Bao, Y. Z. Zhuo, Phys. Lett. A 239 (1998) 228.
- [12] B. ten Hagen, R. Wittkowski, H. LLöwen, Phys. Rev. E 84 (2011) 031105.
- [13] L. Angelani, A. Costanzo, and R. Di Leonardo, EPL 96 (2011) 68002;

- [14] L. Angelani, R. DiLeonardo, and G. Ruocco, Phys. Rev. Lett. 102 (2009) 048104.
- [15] M. B. Wan, C. J. Olson Reichhardt, Z. Nussinov, and C. Reichhardt, Phys. Rev. Lett. 101 (2008) 018102.
- [16] P. K. Ghosh, V. R. Misko, F. Marchesoni, and F. Nori, Phys. Rev. Lett. 110 (2013) 268301.
- [17] A. Pototsky, A. M. Hahn, and H. Stark, Phys. Rev. E 87 (2013) 042124.
- [18] B. ten Hagen, S. van Teeffelen, H. Löwen, J. Phys.: Condensed Matter 23 (2011) 194119.
- [19] Y. Fily and M. C. Marchetti, Phys. Rev. Lett. 108 (2012) 235702.
- [20] S. Henkes, Y. Fily, and M.C. Marchetti, Phys. Rev. E 84 (2011) 040301.
- [21] J. D. Bao, Phys. Rev. E 63 (2001) 061112.
- [22] B. Ai, Q. Chen, Y. He, F. Li, and W. Zhong, Phys. Rev. E 88 (2014) 062129.